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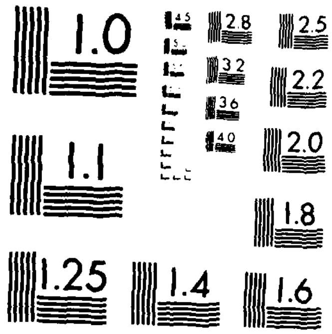

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SLOW TIME-SCALE SOURCE-FREE MAXWELL EQUATIONS FOR A NONSTATIONARY, INHOMOGENEOUS MEDIUM

BY ROBERT CAWLEY

RESEARCH AND TECHNOLOGY DEPARTMENT

FEBRUARY 1983

NSWC

NAVAL SURFACE WEAPONS CENTER

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Source-free Maxwell equations are derived for an electromagnetic (carrier) wave in an inhomogeneous medium exhibiting nonstationary linear response. The non-stationarity is assumed to be slow in comparison with the carrier oscillation, and the equations that result assume that time-derivatives of all slowly varying quantities higher than the first order can be neglected. The equations form the theoretical basis of analysis of the nonlinear dynamics of the coupling of laser energy into a fluid, for the case of laser intensity below threshold for air (plasma) breakdown.

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FOREWORD

Source-free Maxwell equations are derived for an electromagnetic (carrier) wave in an inhomogeneous medium exhibiting nonstationary linear response. The non-stationarity is assumed to be slow in comparison with the carrier oscillation, and the equations that result assume that time-derivatives of all slowly varying quantities higher than the first order can be neglected. The equations form the theoretical basis of analysis of the nonlinear dynamics of the coupling of laser energy into a fluid, for the case of laser intensity below threshold for air (plasma) breakdown.

Approved by:

IRA M. BLATSTEIN, Head
Radiation Division

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CHAPTER 1

INTRODUCTION

The physics of laser induced sound in liquids is divided into two parts, the coupling of the electromagnetic energy of the light field into the fluid to create a disturbed region, from which acoustic energy is radiated, and the propagation, subsequently, of the radiated acoustic wave. In this report, I present a brief examination of a nonlinear aspect of the first of these problems, for the case of surface absorption.

Broadly, two principal coupling mechanisms of laser energy are absorption by particulate impurities in the liquid, and absorption by dielectric loss. Depending upon laser intensities and wavelength, dielectric loss in the fluid may also entail dielectric breakdown. Particulate absorption[1] and breakdown[2] each has been observed in focal regions beneath the liquid surface, for water using ruby lasers ($\lambda = 0.6943$ μm). Miniature volume explosions result in the production of acoustic energy. Early reported observations of laser-induced sound, again using a ruby laser, appeared consistent with simple linear dielectric absorption below threshold for fluid breakdown, leading to thermal generation of an acoustic stress wave.[3]

Early investigations also proved the utility of Nd:glass ($\lambda = 1.06$ μm) and of CO_2 ($\lambda = 10.6$ μm) lasers; in addition, dyes have been used to control the optical absorption constant.[4] The linear optical absorption length,

$$\alpha^{-1} = \frac{1}{2}\bar{\lambda} n_i \ , \qquad n_i \equiv \text{Im}\ n \tag{1.1}$$

where n is the complex refractive index of the fluid, varies with wavelength λ over as many as ten orders of magnitude in water. Thus in the visible region, α^{-1} varies in water from a meter, or so, at the red end to several tens of meters in the blue-green window, while in the ultra-violet, around 600Å, it falls as low as 100Å(=0.01μm). On the long wavelength side, for Nd:glass laser light at 1.06 μm, $\alpha^{-1} = 6.0$ cm, and it falls precipitously again (though non-monotonically), four orders of magnitude to 11.8 μm, at $\lambda = 10.6$ μm (CO_2).

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[1] Bell, C. E., and Landt, J. A., Appl. Phys. Lett., Vol. 10, 1967, p. 46.

[2] Barnes, P. A., Studies of Laser-Induced Breakdown Phenomena in Water, Ph.D. Thesis, Simon Fraser University, 1969.

[3] Carome, E. F., Moeller, C. E., and Clark, N. A., Appl. Phys. Lett., Vol. 4, 1964, p. 95.

[4] Gournay, Luke S., J. Appl. Phys., Vol. 40, 1966, p. 1322.

Figure 1 shows the behavior of the real and imaginary parts of n as functions of frequency, $f = c/\lambda$, for water.[5] Note the strong absorption band centered at $\lambda = 2.950$ μm, where $\alpha^{-1} \approx 0.840$ μm; see Figure 2 taken from recent reported data in the infrared.[6] From tabulated data in Ref. 6, the refractive index of water at $\lambda = 2.950$ μm is $n = 1.317+0.282i$, and at $\lambda = 10.6$ μm, $n - 1.18+0.075i$.

For small values of α^{-1}, penetration of light into the medium is effectively blocked, and the region of disturbed fluid is confined to a thin surface layer. In the CO_2 laser case, where $\lambda \sim \alpha^{-1} \doteq 10^{-3}$ cm, 3.10 j/cm^2 is the minimum value for time-integrated impulse laser intensity necessary to vaporize one absorption depth of water from room temperature (20°C). The first reported CO_2 laser-induced sound study in water is due to Bunkin, et al[7], who observed surface vaporization. (Vaporization also had been observed previously under other conditions by Gournay.[4]) Numerical calculations were performed subsequently by Feiock and Goodwin,[8] who used a one-dimensional hydrocode for an equilibrium vaporization model, which included an inhomogeneous Beer's Law that assumed proportionality of α to ρ^2, where ρ is the (liquid-vapor) medium density.

In fact, surface vaporization, for modest conditions of laser intensity and time-history, is readily achieved and is explosively sudden. A strong air shock,[9] $P/P_0 = 47$ bar, was formed above the water surface by absorption of a 1.67j CO_2 TEA laser pulse (150 ns FWHM) having a film burn diameter 0.80 cm. The spot diameter on the water surface, as defined by the extent of maximal fluid disturbance (shadowgraph pictures) was 0.68 cm. The corresponding laser fluence (intensity impulse) is $1.67/\frac{\pi}{4} \times 0.68^2 = 4.6$j/cm^2, an upper limit estimate only slightly above the threshold scale value of 3.1 j/cm^2 cited earlier for vaporization of one absorption depth.

The weak coupling case of low-level laser input, where only thermal expansion will occur, but no surface vaporization, gives acoustic pressures proportional to K/C_p, where K is the volume coefficient of expansion and C_p the specific heat of the fluid. For water, $K = 2.1\times10^{-4}(K°)^{-1}$, and $C_p = 4.2$j/g-K°, giving $K/C_p = 4.7\times10^{-12}(\text{cm/sec})^{-2}$. For benzene, the same quantity has the value $62.3\times10^{-12}(\text{cm/sec})^{-2}$, which is an order of magnitude larger. The physics of laser-induced thermoacoustic effects has received extensive treatment in recent (Soviet) literature, and has been reviewed recently by Lyamshev and Sedov.[10]

---

[4] Gournay, Luke S., J. Appl. Phys., Vol. 40, 1966, p. 1322.

[5] Jackson, J. D., Classical Electrodynamics (2nd Ed.) (New York: John Wiley & Sons, Inc., 1975), p. 291.

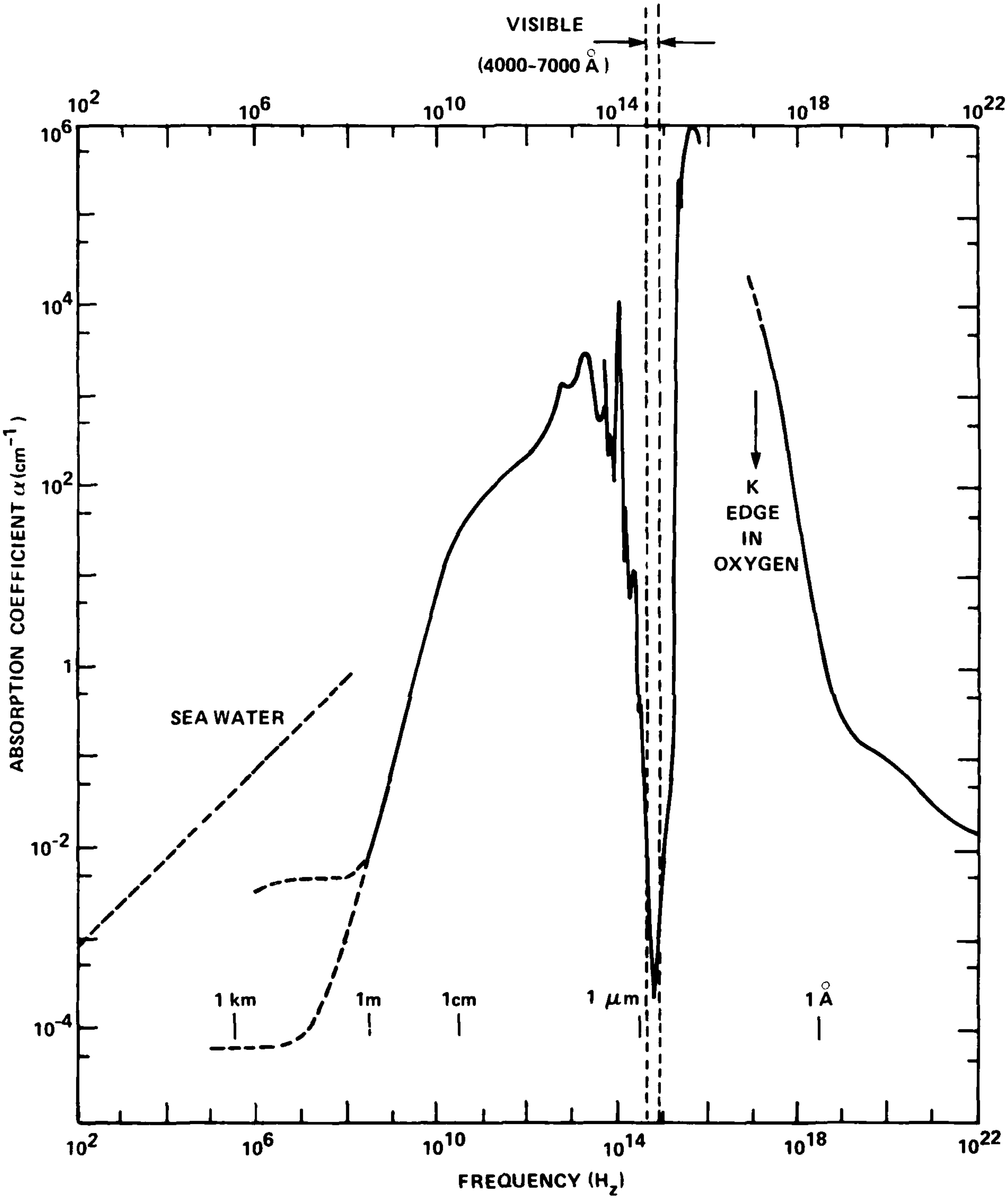
[6] Downing, Harry D., and Williams, Dudley, J. Geophys. Res., Vol. 80, 1975, p. 1956.

[7] Bunkin, F. V., Karlov, N. V., and Komissarov, V. M., Sov. Phys.--JETP Lett., Vol. 13, 1971, p. 341.

[8] Feiock, F. D., and Goodwin, L. K., J. Appl. Phys., Vol. 43, 1972, p. 5061.

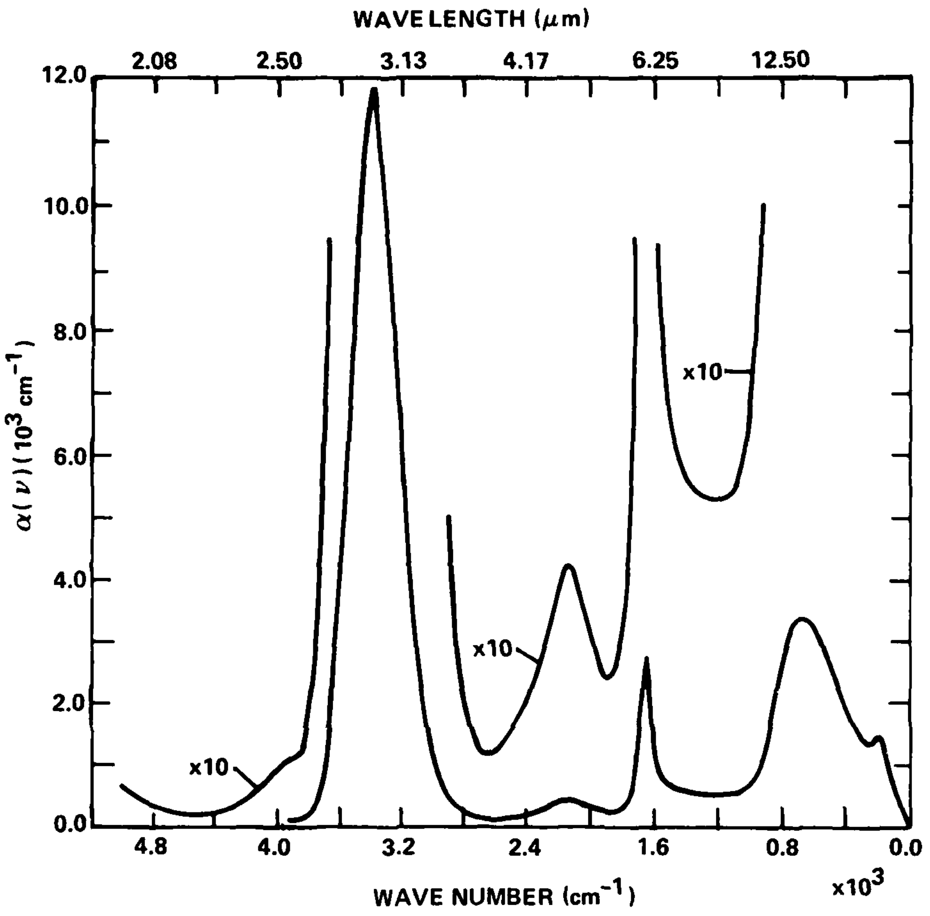
[9] Bell, C. E., and Maccabee, B. S., Applied Optics, Vol. 13, 1974, p. 605.

[10] Lyamshev, L. M., and Sedov, L. V., Sov. Phys.--Acoustics, Vol 27, 1981, p. 4.



NOTE: THE VISIBLE REGION OF THE FREQUENCY SPECTRUM IS INDICATED BY THE VERTICAL DASHED LINES. THE ABSORPTION COEFFICIENT FOR SEA WATER IS INDICATED BY THE DASHED DIAGONAL LINE AT THE LEFT.

FIGURE 1. THE ABSORPTION COEFFICIENT FOR LIQUID WATER AS A FUNCTION OF LINEAR FREQUENCY



NOTE: VALUES SHOWN ARE BASED ON DIRECT MEASUREMENT OF ABSORPTION.

FIGURE 2. LAMBERT ABSORPTION COEFFICIENT α AS A FUNCTION OF WAVE NUMBER AND WAVELENGTH

In contrast, theories of the surface vaporization processes, also reviewed very recently, by Lyamshev and Naugol'nykh'[11] -- who include the topics of nonlinear thermoacoustics and optically induced dielectric breakdown, as well -- are much less well-developed. Two cases (at least) have to be distinguished: (1) High laser intensity, where a plasma is formed above the surface. In the model of Pirri,[12] sensibly all of the energy absorption occurs in the plasma, which forms the leading edge of a detonation front (laser supported detonation wave) that advances in the opposite direction to that of the laser beam propagation. The force delivered to the water surface is the result of shocking the air to high pressure. (2) Low laser intensity, where rapid and sustained vaporization occurs, but no plasma front is created. In this case, the laser energy deposition goes directly into the vapor and the liquid. In both cases, a flow field must be determined in order to predict the full fluid disturbance from which the acoustic pulse is radiated;[13] both the absorption and stress coupling processes in the two cases are different. I will not consider case 1 here, but only case 2.

It is not clear that the flow in the vaporization process is steady since the vaporization layer itself will be no more than a few molecular diameters thick, and is probably unstable.[14] The absorption properties of the vapor and heated water to the incident light will vary with time and with space through the density and temperature dependence of the electromagnetic response functions (dielectric constant), even without a surface instability of the vaporization layer. So, the complete laser field will be affected in the fluid. Except for approximate allowances for dielectric inhomogeneity (e.g., Ref. 8), this aspect of the laser-liquid coupling problem appears not to have received any attention, so far; that aspect is the subject of the present report. The main purpose of an investigation of this kind is to search out and identify mechanisms for optimizing laser energy coupling. Accordingly, I will not restrict the analysis to the case of 10.6 μm radiation in water, exclusively, but will be concerned more generally with nonlinear time-dependent dielectric coupling dynamics, broadly appropriate to the surface evaporation process.

The plan of the report is as follows. In Section 2, I specify the physical basis of the equations to be derived subsequently, which is the constitutive relation for adiabatic non-stationary response of an inhomogeneous, local dielectric. In Section 3, I derive the source-free Maxwell equations, for a medium defined in the previous section. In Section 4, I develop the slow time-scale approximation scheme and derive equations of the evelope fields of a pulse

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[11] Lyamshev, L. M., and Naugol'nykh, K. A., Sov. Phys.--Acoustics, Vol. 27, 1981, p. 357.

[12] Pirri, Anthony N., Phys. of Fluids, Vol. 16, 1973, p. 1435.

[13] Lighthill, M. J., Proc. Roy. Soc. London, Vol A211, 1952, p. 564, and Vol. A222, 1954, p. 1. See also Robert T. Beyer, Nonlinear Acoustics, written for the Naval Sea Systems Command, 1974.

[14] Anisimov, S. I., Tribel'skii, M. I., and Epel'baum, Y. G., Sov. Phys.--JETP, Vol. 51, 1980, p. 802.

wave with a high frequency carrier. In Section 5, I specialize these equations to the case of a one-dimensionally stratified medium, and appropriate to a wave incident normally in the fluid. I also give a short physical discussion of limiting cases, leaving the more detailed analyses needed, for later. In Section 6, I summarize the report, and make a couple of concluding remarks.

CHAPTER 2

CONSTITUTIVE RELATION

The complex dielectric function for water is a function of density p, and of temperature T also. This can be significant in the surface regions especially, and at the vaporization front. Thermal and hydrodynamic response to the laser field create spatial inhomogeneity and time-dependence (non-stationarity) of medium response functions so that the linear (and local)* dielectric displacement is given by

$$D(\vec{x},t) = \int dt' \, \hat{\varepsilon}(\vec{x},t;t-t')E(\vec{x},t'). \tag{2.1}$$

The time-scale for the explicit t-dependence shown for $\hat{\varepsilon}$ is that for variation of p and T, and will be assumed very long and slow in comparison with the laser (carrier) period $f^{-1} = \lambda/c = 0.03$ ps, for $\lambda = 10.6$ μm. If the t-dependence of $\hat{\varepsilon}$ may be regarded as adiabatic, we may write

$$\hat{\varepsilon}(\vec{x},t;\ t-t') \approx \hat{\varepsilon}(\rho,T;\ t-t') \tag{2.2}$$

where the quantity on the right hand side is the response function for a stationary medium, having density and temperature given by the values in the fluid at time t. The explicit (x,t)-dependence required is then assumed to enter through that realized in the dynamics of the fluid for ρ and T; viz. $\rho=\rho(\vec{x},t)$ and $T=T(\vec{x},t)$. Eq. (2.1) now becomes

$$D(\vec{x},t) = \int dt' \hat{\varepsilon}(p(\vec{x},t),T(\vec{x},t);t-t')E(\vec{x},t') \tag{2.3a}$$

$$\equiv \varepsilon_{op}(\rho,T)E, \tag{2.3b}$$

where ε_{op} denotes the integral operator in the first equation. The limitation to the validity of the quasi-static approximation, eqs. (2.2) and (2.3), cannot be assessed without a microscopic (molecular radiator) theory of the laser field absorption process, which is outside the scope of the present analysis from assumptions of classical response.

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*In eq. (2.1), more generally, an integral over $\vec{x}'$ is also present; but for local response, the most general (linear) Kernel $\hat{\varepsilon} = \hat{\varepsilon}(\vec{x},t;\vec{x},t')$ is proportional to $\delta^{(3)}(\vec{x}-\vec{x}')$, which results in eq. (2.1).

For a stationary medium, with ρ,T independent of t, Eq. (2.3) is a convolution, so its Fourier transform with respect to the time is

$$\tilde{D}(\vec{x},\omega) = \varepsilon(\rho,T;\omega)\tilde{E}(\vec{x},\omega), \tag{2.4}$$

with an obvious notation; in particular $\varepsilon(\rho,t;\omega)$ is the Fourier transform of the right side of Eq. (2.2), but with p,T independent of t. When explicit t-dependence from medium response is present in $\hat{\varepsilon}$, Eq. (2.4) does not follow from Eq. (2.3).

Finally, the dc-conductivity σ depends upon the thermodynamic state of the fluid also, $\sigma=\sigma(\rho,T)$, and

$$J(\vec{x},t) = \sigma(\rho(\vec{x},t),\ T(\vec{x},t)(E(\vec{x},t) \tag{2.5}$$

is the quasi-static model analogue of Eq. (2.3).

The electromagnetic fields of the laser wave in the fluid will have a fast and a slow time-scale (envelope) part, and equations for the latter will be derived. Preparatory to attendant labors of this effort I list a few relationships issuing from eq. (2.3). The simplest is

$$\nabla xD = \nabla\varepsilon_{op}(\rho,T)xE + \varepsilon_{op}(\rho,T)\nabla xE, \tag{2.6a}$$

where the meaning of the first term on the right side is

$$\nabla\varepsilon_{op}(\rho,T)xE \equiv \int dt'\nabla\hat{\varepsilon}(\rho(\vec{x},t),T(\vec{x},t),t-t')xE(t'). \tag{2.6b}$$

If E = 0 at $t\to-\infty$, differentiation of eq. (2.3a) gives

$$\frac{\partial D}{\partial t} = \frac{\partial\varepsilon_{op}(\rho,T)}{\partial t}.E + \varepsilon_{op}(\rho,T)\cdot\frac{\partial E}{\partial t}, \tag{2.7a}$$

where

$$\frac{\partial\varepsilon_{op}(\rho,T)}{\partial t}.E \equiv \int dt'\ \frac{\partial\hat{\varepsilon}(\rho(\vec{x},t),T(\vec{x},t);t-t')}{\partial t}E(t') \tag{2.7b}$$

in which t-t' is held fixed for the derivative of $\hat{\varepsilon}$. Various further relations

follow similarly, such as,

$$\frac{\partial}{\partial t}(\nabla\varepsilon_{op} x E) = \frac{\partial\nabla\varepsilon_{op}}{\partial t} x E + \nabla\varepsilon_{op} x \frac{\partial E}{\partial t}. \tag{2.8}$$

In what follows, I will suppress the ρ,T dependence of ε_{op} in the notation, and I will drop the "op" suffix as well.

Finally, I also need the inverse of the integral operator ε, which I define by

$$E(t) \equiv (\varepsilon^{-1} \cdot D)(t) \equiv \int dt' \widehat{\varepsilon^{-1}}(\rho,T;t-t')D(t'). \tag{2.9}$$

Substituting Eq. (2.3a), and denoting t'-dependence by primes on ρ and T,

$$E(t) = \int dt' \int dt'' \widehat{\varepsilon^{-1}}(\rho,T,t-t')\hat{\varepsilon}(\rho',T';t'-t'')E(t'') \tag{2.10}$$

whence, since E is arbitrary,

$$\int dt' \ \widehat{\varepsilon^{-1}}(\rho,T;t-t')\hat{\varepsilon}(\rho',T';t'-t'') = \delta(t-t''). \tag{2.11a}$$

Similarly, starting from D = εE, one finds

$$\int dt' \hat{\varepsilon}(\rho,T;t-t')\widehat{\varepsilon^{-1}}(\rho',T';t'-t'') = \delta(t-t''). \tag{2.11b}$$

Notice that

$$\int dt' \widehat{\varepsilon^{-1}}(\rho,T;t-t')\hat{\varepsilon}(\rho,T;t'-t'') \neq \delta(t-t'') \tag{2.12}$$

unless $\partial\rho/\partial t = 0$ and $\partial T/\partial t = 0$; this parallels the remark made earlier concerning Eq. (2.4). Notice also that the left sides of Eqs. (2.11) are not convolutions.

Note, finally, that Eq. (2.11), which I may write more economically as

$$\varepsilon^{-1} \varepsilon = 1_{op}, \tag{2.13}$$

implies

$$\nabla\varepsilon^{-1} \cdot \varepsilon \equiv -\varepsilon^{-1} \cdot \nabla\varepsilon, \tag{2.14}$$

since $\delta(t-t'')$ has no $\vec{x}$-dependence. In the same way,

$$\nabla\varepsilon \cdot \varepsilon^{-1} \equiv -\varepsilon \cdot \nabla\varepsilon^{-1}. \tag{2.15}$$

CHAPTER 3

FIELD EQUATIONS

Taking the curl of

$$\nabla x E = -\frac{1}{c}\frac{\partial B}{\partial t}, \tag{3.1}$$

and using

$$\nabla x B = \frac{4\pi\sigma}{c} E + \frac{1}{c}\frac{\partial D}{\partial t}, \tag{3.2}$$

gives

$$\nabla^2 E - \frac{4\pi}{c^2}\frac{\partial}{\partial t}(\sigma E) - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = \nabla(\nabla \cdot E), \tag{3.3}$$

Here $\nabla x \nabla x \equiv -\nabla^2 + \nabla(\nabla \cdot)$ has been used. Similarly, Eq. (3.2), together with $\nabla \cdot B = 0$, gives

$$\nabla^2 B = -\frac{4\pi}{c}\nabla x(\sigma E) - \frac{1}{c}\frac{\partial}{\partial t}(\nabla x D). \tag{3.4}$$

To throw Eq. (3.4) into a wave equation form, I have to manipulate the last term, making liberal use of Eqs. (2.3) - (2.9). Using Eq. (3.1), Eq. (3.4)

$$\nabla^2 B = -\frac{4\pi}{c}\nabla x(\sigma E) - \frac{1}{c}\frac{\partial}{\partial t}(\nabla\varepsilon x E + \varepsilon\nabla x E), \tag{3.5}$$

reads

$$\nabla^2 B - \frac{\varepsilon}{c^2}\frac{\partial^2 B}{\partial t^2} = -\frac{4\pi}{c}\nabla x(\sigma E) - \frac{1}{c}\frac{\partial\nabla\varepsilon}{\partial t} xE - \frac{1}{c}\nabla\varepsilon x\frac{\partial E}{\partial t} + \frac{1}{c^2}\frac{\partial\varepsilon}{\partial t}\frac{\partial B}{\partial t}, \tag{3.6}$$

For the third term on the right, I use Eq. (3.2), with Eq. (2.3), to write

$$\frac{1}{c}\nabla\varepsilon x\frac{\partial E}{\partial t} \equiv \nabla\varepsilon x\varepsilon^{-1}\left(\frac{\varepsilon}{c}\frac{\partial E}{\partial t}\right) \equiv \left(\nabla\varepsilon\cdot\ \varepsilon^{-1}\right)x\left(\frac{\varepsilon}{c}\frac{\partial E}{\partial t}\right)$$

$$= \nabla\varepsilon\cdot\varepsilon^{-1}\ x\left(\nabla xB - \frac{4\pi\sigma}{c}E - \frac{1}{c}\frac{\partial\varepsilon}{\partial t}E\right); \tag{3.7}$$

while for the first, I use Eq. (3.1), and have

$$\frac{4\pi}{c}\nabla x(\sigma E) = \frac{4\pi}{c}\nabla\sigma xE - \frac{4\pi\sigma}{c^2}\frac{\partial B}{\partial t} \tag{3.8}$$

Substituting these into Eq. (3.6), one arrives at

$$\left(\nabla^2 - \frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)B + \left(\nabla\varepsilon\cdot\varepsilon^{-1}\right)x\nabla xB - \frac{1}{c^2}\left(\frac{\partial\varepsilon}{\partial t} + 4\pi\sigma\right)\frac{\partial B}{\partial t}$$

$$= -\frac{1}{c}\nabla\left(4\pi\sigma + \frac{\partial\varepsilon}{\partial t}\right)xE + \frac{1}{c}\nabla\varepsilon\cdot\varepsilon^{-1}\left(4\pi\sigma + \frac{\partial\varepsilon}{\partial t}\right)xE \tag{3.9}$$

A somewhat less involved treatment of Eq. (3.3), but along the same lines, gives

$$\left(\nabla^2 - \frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)E - \frac{1}{c^2}\left(2\frac{\partial\varepsilon}{\partial t} + 4\pi\sigma\right)\frac{\partial E}{\partial t} - \nabla(\nabla\cdot E)$$

$$= \frac{4\pi}{c^2}\frac{\partial\sigma}{\partial t}E + \frac{1}{c^2}\frac{\partial^2\varepsilon}{\partial t^2}E. \tag{3.10}$$

Eqs. (3.9) and (3.10) are exact consequences of Maxwell's equation for a nonstationary inhomoheneous medium characterized by constitutive relations defined by Eqs. (2.3) and (2.5). The special case $\sigma \to 0$ and $\partial\varepsilon/\partial t \to 0$ gives the field equations,

$$\left(\nabla^2 - \frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)B_o + \left(\varepsilon^{-1}\nabla\varepsilon\right)x\left(\nabla x B_o\right) = 0 \qquad (3.11)$$

$$\left(\nabla^2 - \frac{\varepsilon}{c^2}\frac{\partial^2}{t^2}\right)E_o - \nabla\left(\nabla \cdot E_o\right) = 0, \qquad (3.12)$$

which agrees with standard textbook results for stationary media.[15]

[15] Landau, L. D., and Lifshitz, E. M., Electrodynamics of Continuous Media, (Reading, Massachusetts: Addison-Wesley, 1960), p. 284ff.

CHAPTER 4

SLOW TIME-SCALE FIELD EQUATIONS

I denote the laser frequency now by ω, and write the fields as

$$E = \mathrm{Re}(\mathcal{E}\ \exp{-i\omega t}) \tag{4.1}$$

$$B = \mathrm{Re}(\mathcal{B}\ \exp{-i\omega t}), \tag{4.2}$$

where $\mathcal{E}$ and $\mathcal{B}$ are spatially and (slowly) time-varying complex envelope fields. In Eqs. (3.9) and (3.10) it is necessary to evaluate several integral operator expressions, such as

$$\frac{\partial^2 \varepsilon}{\partial t^2} E = \mathrm{Re}\int dt' \, \frac{\partial^2 \hat{\varepsilon}(\rho,T;t-t')}{\partial t^2} e^{-i\omega t'} \, \mathcal{E}(t'), \tag{4.3}$$

where I have suppressed the $\vec{x}$-dependence of $\mathcal{E}$. The right side of Eq. (4.3) has the general form

$$\mathrm{Re}\left[e^{-i\omega t}\int d\tau \hat{h}(\rho,T;\tau) e^{i\omega\tau} \, \mathcal{E}(t-\tau)\right], \tag{4.4}$$

where I have changed variables with $t' \equiv t-\tau$. Expanding $\mathcal{E}$ about $\tau=0$ gives the quantity in the square bracket formally, with $\mathcal{E}^{(n)}(t) \equiv \partial^n \mathcal{E}/\partial t^n$, as

$$e^{-i\omega t} \sum_{n=o}^{\infty} \frac{E^{(n)}(t)}{n!} \int d\tau (-\tau)^n e^{i\omega\tau} \hat{h}(\rho,T;\tau)$$

$$= e^{-i\omega t} \sum_{n=o}^{\infty} \frac{i^n}{n!} E^{(n)}(t) \frac{d^n}{d\omega^n} \int d\tau e^{i\omega\tau} \hat{h}(\rho,T;\tau)$$

$$= e^{-i\omega t} \left[E(t) h(\rho,T;\omega) + \frac{i}{\omega} \frac{\partial E}{\partial t} \cdot \omega \frac{dh(\rho,T;\omega)}{d\omega} + \ldots \right], \tag{4.5}$$

where h is the Fourier transform of $\hat{h}$. Thus, for Eq. (4.3),

$$h = h\ (\rho,T;\omega) \equiv \frac{\partial^2 \varepsilon(\rho,T;\omega)}{\partial t^2}\ . \tag{4.6}$$

In Eq. (4.5) the ratio of the second term to the first is of order $(\omega\tilde{t})^{-1}$, where $\tilde{t}$ is a time-scale for significant variation of E; since this ratio is extremely small for the adiabatic conditions of the problem, only the first term has to be retained. In this way, Eq. (4.3) becomes, to very good approximation,

$$\frac{\partial^2 \varepsilon}{\partial t^2} E \approx \mathrm{Re} \left[e^{-i\omega t} \frac{\partial^2 \varepsilon(\omega)}{\partial t^2} E \right], \tag{4.7}$$

where I have written $\varepsilon(\omega)$ for $\varepsilon(\rho,T;\omega)$ to simplify notation.

Similarly, the term containing $\partial E/\partial t$ in Eq. (3.10) involves an expression of the form

$$\int dt'\hat{h}(\rho,T;t-t')\frac{\partial}{\partial t'}\left(e^{-i\omega t'}E(t')\right)$$

$$= e^{-i\omega t}\int d\tau\hat{h}(\rho,T;\tau)e^{i\omega\tau}\left(-i\omega+\frac{\partial}{\partial t}\right)E(t-\tau)$$

$$= e^{-i\omega\tau}\sum_{n=o}^{\infty}\frac{i^n}{n!}\left(-i\omega+\frac{\partial}{\partial t}\right)E^{(n)}(t)\ \frac{d^n h(\rho,T;\omega)}{d\omega^n}\ , \qquad (4.8)$$

where now $h = \partial\varepsilon/\partial t$, while the $\partial^2E/\partial t^2$-term involves

$$e^{-i\omega t}\sum_{n=o}^{\infty}\frac{i^n}{n!}\left(-\omega^2-2i\omega\frac{\partial}{\partial t}+\frac{\partial^2}{\partial t^2}\right)E^{(n)}(t)\ \frac{d^n h(\rho,T;\omega)}{d\omega^n}\ , \qquad (4.9)$$

in which $h = \varepsilon$. The lowest terms in Eq. (4.8), out to the same order as the $\partial^2\varepsilon/\partial t^2$ -term in Eq. (4.7), include the $n = 1$ coefficient of $-i\omega$, so that

$$\frac{\partial\varepsilon}{\partial t}\frac{\partial E}{\partial t} = \left\{\mathrm{Re}\ e^{-i\omega t}\left[-i\omega\,\frac{\partial\varepsilon(\omega)}{\partial t} + \left(1-i\omega\frac{d}{d\omega}\right)\frac{\partial\varepsilon(\omega)}{\partial t}\frac{\partial E}{\partial t}\right]\right\}, \qquad (4.10)$$

while, again to the same order, Eq. (4.9) gives

$$\varepsilon\frac{\partial^2E}{\partial t^2} = \mathrm{Re}\left\{e^{-i\omega t}\left[-\omega^2\varepsilon(\omega)E - 2i\omega\left(1+\frac{1}{2}\,\omega\frac{d}{d\omega}\right)\varepsilon(\omega)\frac{\partial E}{\partial t}\right.\right.$$

$$\left.\left. + \left(1 + 2\omega\frac{d}{d\omega} + \frac{1}{2}\omega^2\frac{d^2}{d\omega^2}\right)\varepsilon(\omega)\frac{\partial^2E}{\partial t^2}\right]\right\}. \qquad (4.11)$$

I neglect terms involving the second time derivatives of adiabatically varying quantities from here on. Collecting results, Eq. (3.10) finally reduces to*

$$\left[\nabla^2 + \frac{\omega^2}{c^2}\,\varepsilon^{(E)}(\omega)\right]E - \nabla(\nabla\cdot E)$$

$$+ \frac{2i\omega}{c^2}\left(1 + \frac{\omega}{2}\frac{d}{d\omega}\right)\varepsilon(\omega)\,\frac{\partial E}{\partial t} = 0 \qquad (4.12)$$

where

$$\varepsilon^{(E)}(\omega) = \varepsilon(\omega) + \frac{i}{\omega}\,(4\pi\sigma + 2\,\frac{\partial\varepsilon}{\partial t}) - \frac{4\pi}{\omega^2}\frac{\partial\sigma}{\partial t}. \qquad (4.13)$$

Three of the terms in Eq. (3.9) for the magnetic field involve compositions of integral operators; in addition, they all involve ε^{-1}. Each of these features results in further complications requiring special attention. The term containing ∇xB, on the left side, involves

$$(\nabla\varepsilon\cdot\varepsilon^{-1})\ x\ \mathrm{curl}\ B = \mathrm{Re}\int dt''(\nabla\varepsilon\cdot\varepsilon^{-1})\ (t,t'')$$

$$x\ \mathrm{curl}\ e^{-i\omega t''}B\ (t''), \qquad (4.14)$$

*Eq. (3.10) asserts the vanishing of an expression of the form $R(e^{-i\omega t}F)$, where F is the left side of Eq. (4.12). Since F is sensibly constant for variations of t on the scale of ω^{-1}, this condition evaluated at $t = 0$ and $t = \pi/2\omega$ implies the separate vanishing of real and imaginary parts of F, whence follows Eq. (4.12).

where

$$\nabla\varepsilon\ \varepsilon^{-1}(t,t'') \equiv \int dt' \nabla\hat{\varepsilon}\ (\rho,T;t-t')\ \widehat{\varepsilon^{-1}}(\rho',T';t'-t'') \qquad (4.15)$$

Proceeding as before, with the variable change $t''=t-\tau$, and with the Taylor series expansion for $\mathcal{B}(t-\tau)$,

$$\begin{aligned}(\nabla\varepsilon\ \varepsilon^{-1}) \times \text{curl}\ \mathcal{B} &= \text{Re}\left[e^{-i\omega t}\int d\tau(\nabla\varepsilon\ \varepsilon^{-1})(t,t-\tau)e^{i\omega\tau} \times \text{curl}\ \mathcal{B}(t-\tau)\right] \\ &= \text{Re}\left[e^{-i\omega t}\sum_{n=0}^{\infty}\frac{i^n}{n!}\frac{d^n}{d\omega^n}\left(\int d\tau e^{i\omega\tau}\ (\nabla\varepsilon\ \varepsilon^{-1})(t,t-\tau)\right)\right. \\ &\qquad \left. \times\ \text{curl}\ \mathcal{B}^{(n)}\ (t)\right]. \end{aligned} \qquad (4.16)$$

Substituting Eq. (4.15), with some arranging of the arguments of the exponential, and of ρ', T',

$$\int d\tau\, e^{i\omega\tau}(\nabla\varepsilon\cdot\varepsilon^{-1})(t,t-\tau) = \int d\tau \int dt'\, e^{i\omega(t-t')}\, \nabla\hat{\varepsilon}(\rho,T;t-t')$$

$$e^{i\omega(t'-t+\tau)}\widehat{\varepsilon^{-1}}\left(\rho(t-(t-t'),T(t-(t-t'));\ t'-t+\tau\right)$$

$$= \int dt_1\, e^{i\omega t_1}\nabla\hat{\varepsilon}\,(\rho,T;t_1)\int d\tau_1\, e^{i\omega t_1}$$

$$\widehat{\varepsilon^{-1}}'\left(\rho(t-t_1),T(t-t_1);\tau_1\right)$$

$$= \sum_{n=0}^{\infty} \frac{i^n}{n!}\frac{d^n}{d\omega^n}\left(\int dt_1 e^{i\omega t_1}\nabla\hat{\varepsilon}(\rho,T;t_1)\right)$$

$$\times\frac{\partial^n}{\partial t^n}\left(\int d\tau_1 e^{i\omega t_1}\widehat{\varepsilon^{-1}}(\rho,T;\tau_1)\right) \tag{4.17}$$

where in the last line I have performed a Taylor expansion again, this time of $\widehat{\varepsilon^{-1}}$ about $t_1 = 0$. If I now adopt the notation

$$\int d\tau_1 e^{i\omega\tau_1}\ \widehat{\varepsilon^{-1}}\left(\rho,T;\tau_1\right) \equiv \varepsilon^{-1}(\omega), \tag{4.18}$$

Eq. (4.17) takes the more economical form

$$\int d\tau e^{i\omega\tau} \, (\nabla\varepsilon . \varepsilon^{-1})(t,t-\tau)$$

$$= \sum_{n=0}^{\infty} \frac{i^n}{n!} \frac{d^n \nabla\varepsilon(\omega)}{d\omega^n} \frac{\partial^n \varepsilon^{-1}(\omega)}{\partial t^n}. \tag{4.19}$$

As noted earlier, Eqs. (2.11) do not have the form of convolutions, a consequence of which is that $\varepsilon^{-1}(\omega) \neq 1/\varepsilon(\omega)$, as would be the case for stationary medium response. Starting from Eq. (2.11a), with $t'' = t-\tau$, the analysis leading to Eq. (4.19) can be applied, mutatis mutandis, to give

$$1 = \sum_{n=0}^{\infty} \frac{i^n}{n!} \frac{d^n \varepsilon^{-1}(\omega)}{d\omega^n} \frac{\partial^n \varepsilon(\omega)}{\partial t^n}, \tag{4.20}$$

from which a formal expression of $\varepsilon^{-1}(\omega)$ can be constructed recursively. In the lowest order, adiabatic approximation, Eq. (4.20) reads

$$\varepsilon^{-1}(\omega)\varepsilon(\omega) \doteq 1 - i \frac{d\varepsilon^{-1}(\omega)}{d\omega} . \frac{\partial \varepsilon(\omega)}{\partial t} , \tag{4.21}$$

so that

$$\varepsilon^{-1}(\omega) \doteq \frac{1}{\varepsilon(\omega)} - \frac{i}{\varepsilon(\omega)} \frac{d}{d\omega} \left(\frac{1}{\varepsilon(\omega)} \right) \frac{\partial \varepsilon(\omega)}{\partial t} , \tag{4.22}$$

which is sufficient for purposes of the present problem. Eq. (4.20) and Eq. (4.22) both reduce to the stationary medium result in the limit of $\varepsilon(\omega)$ independent of t.

combining Eq. (4.19) and (4.22), one has

$$\int d\tau e^{i\omega\tau} \; (\nabla\varepsilon\cdot\varepsilon^{-1})(t,t-\tau) \doteq \left[\frac{1}{\varepsilon(\omega)} - \frac{i}{\varepsilon(\omega)}\frac{d}{d\omega}\left(\frac{1}{\varepsilon(\omega)}\right)\frac{\partial\,\varepsilon(\omega)}{\partial t}\right] \nabla\varepsilon(\omega)$$

$$+\; i\,\frac{\partial}{\partial t}\left(\frac{1}{\varepsilon(\omega)}\right)\frac{d}{d\omega}\,\nabla\varepsilon(\omega), \qquad (4.23)$$

for use in Eq. (4.16); substituting, one arrives, after a little algebra, at

$$\left(\nabla\varepsilon\cdot\varepsilon^{-1}\right) x \text{ curl } B \doteq \mathrm{Re}\Bigg\{e^{-i\omega t}\Bigg[\frac{1}{\varepsilon(\omega)}\nabla\varepsilon(\omega) x \text{ curl } \mathcal{B}(t)$$

$$-i\;\frac{\partial \log\,\varepsilon(\omega)}{\partial\, t}\,\frac{d}{d\omega}\,\nabla\,\log\,\varepsilon(\omega)\;\; x \text{ curl } \mathcal{B}\;(t)$$

$$+\; i\,\frac{d}{d\omega}\Big(\nabla\;\log\;\varepsilon(\omega)\Big)\;\; x \text{ curl } \frac{\partial\mathcal{B}}{\partial\, t}\Bigg]\Bigg\} \qquad (4.24)$$

That leaves the last two of the four terms on the right side of Eq. (3.9). The first of these is similar to (4.16),

$$\left(\nabla\varepsilon\cdot\varepsilon^{-1}\right) x\sigma E = \mathrm{Re}\left[e^{-i\omega t}\int d\tau\;\nabla\varepsilon\cdot\varepsilon^{-1}\;(t,t-\tau)e^{i\omega\tau}\; x\sigma(t-\tau)\mathcal{E}(t-\tau)\right]$$

$$= \mathrm{Re}\left[e^{-i\omega t}\sum_{n=0}^{\infty}\frac{i^n}{n!}\,\frac{d^n}{d\omega^n}\left(\int d\tau e^{i\omega\tau}(\nabla\varepsilon.\varepsilon^{-1})(t,t-\tau)\right)\right.$$

$$\left. x\;\frac{\partial^n(\sigma\mathcal{E})}{\partial t^n}\right], \qquad (4.25)$$

whence, with the aid of Eqs. (4.19) and (4.22),

$$\left(\nabla\varepsilon\cdot\varepsilon^{-1}\right)\mathrm{x}\sigma E \doteq \mathrm{Re}\left\{e^{-i\omega t}\left[\frac{\sigma}{\varepsilon(\omega)}\nabla\varepsilon(\omega)\mathrm{x}E\right.\right.$$

$$\left.\left.-i\sigma\,\frac{\partial\log\,\varepsilon(\omega)}{\partial t}\frac{d}{d\omega}\left(\nabla\log\,\varepsilon(\omega)\right)\mathrm{x}E + i\frac{d}{d\omega}\left(\nabla\log\,\varepsilon(\omega)\right)\mathrm{x}\,\frac{\partial}{\partial t}(\sigma E)\right]\right\} \tag{4.26}$$

The remaining term contains a product of three operators; but it is also already of first order smallness in the adiabatic regime. One has

$$\nabla\varepsilon\cdot\varepsilon^{-1}\cdot\frac{\partial\,\varepsilon}{\partial\,t}\mathrm{x}E = \mathrm{Re}\left[\,e^{-i\omega t}\int d\tau e^{\,i\omega\tau}\left(\nabla\varepsilon\cdot\varepsilon^{-1}\cdot\frac{\partial\,\varepsilon}{\partial\,t}\right)(t,t-\tau)\right.$$

$$\left.\mathrm{x}E(t) + \ldots\right], \tag{4.27}$$

with

$$\int d\tau e^{i\omega\tau}\left(\nabla\varepsilon\cdot\varepsilon^{-1}\cdot\frac{\partial\,\varepsilon}{\partial\,t}\right)(t,t-\tau)$$

$$= \int dt_1\, e^{\,i\omega t_1}\left(\nabla\varepsilon\cdot\varepsilon^{-1}\right)(t,t-t_1)\;\cdot\frac{\partial\,\varepsilon(\omega)}{\partial\,t}$$

$$= \frac{1}{\varepsilon(\omega)}\;\nabla\varepsilon(\omega)\;\frac{\partial\,\varepsilon(\omega)}{\partial\,t} + \ldots, \tag{4.28}$$

so that

$$\nabla\varepsilon\cdot\varepsilon^{-1}\cdot\frac{\partial\varepsilon}{\partial t}\times E \doteq \mathrm{Re}\left[e^{-i\omega t}\frac{\partial\varepsilon(\omega)}{\partial t}\cdot\nabla\log\,\varepsilon(\omega)\times\mathcal{E}\,(t)\right]. \tag{4.29}$$

The equation to be satisfied by $\mathcal{B}(t)$, analogous to Eq. (4.12) for $\mathcal{E}(t)$, can be written down directly after substituting Eq. (4.2) into Eq. (3.9) by making use of the foregoing results, viz. Eqs. (4.11), (4.24), (4.10), Eq. (4.5) with $h=\partial\varepsilon(\omega)/\partial t$, and Eqs. (4.26) and (4.29). That equation is

$$\left[\nabla^2+\frac{\omega^2}{c^2}\varepsilon^{(B)}(\omega)\right]\mathcal{B}+\left(1-i\frac{\partial\log\,\varepsilon(\omega)}{\partial t}\frac{d}{d\omega}\right)\nabla\log\,\varepsilon(\omega)\times\mathrm{curl}\,\mathcal{B}$$

$$+\,i\frac{d}{d\omega}\left(\nabla\log\,\varepsilon(\omega)\right)\times\mathrm{curl}\,\frac{\partial\mathcal{B}}{\partial t}+\frac{2i\omega}{c^2}\left(1+\frac{\omega}{2}\frac{d}{d\omega}\right)\varepsilon(\omega)\cdot\frac{\partial\mathcal{B}}{\partial t}$$

$$=a(\varepsilon,\sigma)\times\mathcal{E}+b(\varepsilon,\sigma)\times\frac{\partial\mathcal{E}}{\partial t} \tag{4.30}$$

where

$$\varepsilon^{(B)}(\omega)=\varepsilon(\omega)+\frac{i}{\omega}\left(4\pi\sigma+\frac{\partial\varepsilon(\omega)}{\partial t}\right) \tag{4.31a}$$

$$a(\varepsilon,\sigma)=-\frac{4\pi}{c}\nabla\sigma+\frac{\sigma}{c}\left(1+i\frac{\partial\,\log\,\varepsilon(\omega)}{\partial t}\frac{d}{d\omega}\right)\nabla\log\,\varepsilon(\omega)$$

$$+\,i\frac{\partial\sigma}{\partial t}\frac{d}{d\omega}\nabla\log\,\varepsilon\,(\omega)-\frac{\varepsilon}{c}\frac{\partial}{\partial t}\nabla\,\log\,\varepsilon(\omega) \tag{4.31b}$$

$$b(\varepsilon,\sigma)=i\frac{\sigma}{c}\frac{d}{d\omega}\nabla\,\log\,\varepsilon(\omega). \tag{4.31c}$$

<u>Eqs. (4.12) and (4.30) are the slow time-scale equations of the wave. No approximations have been made regarding spatially varying properties of the medium.</u>

CHAPTER 5

ONE-DIMENSIONAL MODEL

I assume a one-dimensionally stratified fluid now, for which ε depends significantly upon only one of the three spatial variables, z, so that

$$\nabla\varepsilon = \hat{z}\,\frac{\partial\varepsilon}{\partial z}\,, \tag{5.1}$$

where $\hat{z}$ is a unit-vector along the positive $\hat{z}$ - axis (downward into the fluid, let us say.) Eqs. (4.12) and (4.30) are consistent in this case with vanishing $\hat{z}$ - components of the field amplitudes. Assuming $\mathcal{E}_z = 0$, one can derive $\nabla\cdot\mathcal{E} = 0$ from $\nabla\cdot\mathcal{D} = 0$; one has

$$\nabla\cdot(\varepsilon E) = \nabla\varepsilon\cdot E + \varepsilon\,\nabla\cdot E = 0 \tag{5.2}$$

so that

$$\nabla\cdot E = -\varepsilon^{-1}\nabla\varepsilon\cdot E = 0,\quad E_z = 0. \tag{5.3}$$

From Eq. (5.3) and Eq. (4.1), Eq. (4.12) then becomes

$$\left[\nabla^2 + \frac{\omega^2}{c^2}\,\varepsilon^{(E)}(\omega)\right]\mathcal{E}_x + \frac{\partial i\omega}{c^2}\left(1 + \frac{\omega}{2}\frac{d}{d\omega}\right)\varepsilon\,(\omega)\,\frac{\partial\mathcal{E}_x}{\partial t} = 0, \tag{5.4}$$

with a similar equation for $\mathcal{E}_y$. I consider here only the case that the direction of $\mathcal{E}$ is constant and uniform, however, which I take now to be the $\hat{x}$-axis,

$$\mathcal{E} = \mathcal{E}_x\hat{x},\ \mathcal{E}_y = 0. \tag{5.5}$$

Then Eq. (4.30) supports solutions,

$$\mathcal{B} = \mathcal{B}_y\ \hat{y}\ , \qquad \hat{y} = \hat{z} x \hat{x}. \tag{5.6}$$

From $\mathcal{B}_z = 0$,

$$\nabla \log \varepsilon(\omega) \times \operatorname{curl} \mathcal{B} = - \frac{\partial \log \varepsilon(\omega)}{\partial Z} \frac{\partial \mathcal{B}_y}{\partial Z}\ , \tag{5.7}$$

with a similar equation for the curl $(\partial \mathcal{B}/\partial t)$ - term in (4.30). The equation that results for $\mathcal{B}_y$ is

$$\left[\nabla^2 - \left(1 - i \frac{\partial \log \varepsilon(\omega)}{\partial t} \frac{d}{d\omega}\right) \frac{\partial \log \varepsilon(\omega)}{\partial z} \frac{\partial}{\partial z} + \frac{\omega^2}{c^2} \varepsilon^{(B)}(\omega)\right] \mathcal{B}y$$

$$-i \frac{d}{d\omega}\left(\frac{\partial \log \varepsilon(\omega)}{\partial z}\right) \frac{\partial^2 \mathcal{B}y}{\partial z \partial t} + \frac{2 i\omega}{c^2} \left(1 + \frac{\omega}{2} \frac{d}{d\omega}\right) \varepsilon(\omega) \frac{\partial \mathcal{B}y}{\partial t}$$

$$= a(\varepsilon,\sigma) + b\ (\varepsilon,\sigma)\ E_x. \tag{5.8}$$

Eq. (5.4) for E_x is readily interpreted. Its time-independent form, with also $\sigma = 0$, is

$$\left(\nabla^2 + \frac{\omega^2}{c^2} \varepsilon(\omega)\right) E_x = 0, \tag{5.9}$$

by Eq. (4.13). This is the usual time-independent "Schrödinger" equation for propagation in an inhomogeneous medium, with ε playing the role of (optical) potential. The eikonal, or WKB approximation to Eq. (5.9), for the case $\nabla^2 \to \partial^2/\partial z^2$, is

$$E_x = E_{x0} \exp\left(ik_o \int_0^z dz' \sqrt{\varepsilon(\rho(z'),T(z');\omega)}\right) \tag{5.10}$$

where $k_o \equiv \omega/c$ and E_{xo} is a constant, and where the positive square root (forward propagating, damped solution) is understood. When the WKB approximation is invalid, which is the case of significant z-dependence of ε on the scale of $c/\omega = k_o^{-1} \equiv \lambda\!\!\!^{-}$, the solution, Eq. (5.10), no longer holds, and Eq. (5.9) has to be solved explicitly. Finally, in Eq. (5.4), the time derivative term reflects a wave dynamical effect associated with medium non-stationarity.

Eq. (5.8) for $\mathcal{B}_y$ is more complicated, a situation also true in the time-independent, stationary medium limit. In the latter case, with also $\sigma = 0$ again, Eq. (5.8) becomes

$$\left(\nabla^2 - \frac{\partial \log \varepsilon(\omega)}{\partial z} \frac{\partial}{\partial z} + \frac{\omega^2}{c^2} \varepsilon(\omega)\right) \mathcal{B}_y = 0. \tag{5.11}$$

The substitution,

$$\mathcal{B}_y = \sqrt{\varepsilon(\omega)}\, \tilde{\mathcal{B}}_y, \tag{5.12}$$

eliminates the $\partial/\partial z$ terms from (5.11) giving

$$\left(\nabla^2 + \frac{\omega^2}{c^2} \tilde{\varepsilon}(\omega)\right) \tilde{\mathcal{B}}_y = 0, \tag{5.13}$$

the same form as Eq. (5.9), but with now

$$\tilde{\varepsilon}(\omega) = \varepsilon(\omega) + \frac{c^2}{\omega^2}\left[\frac{1}{2} \frac{\partial^2 \log \varepsilon(\omega)}{\partial z^2} - \frac{1}{4}\left(\frac{\partial \log \varepsilon(\omega)}{\partial z}\right)^2\right] \tag{5.14}$$

This recovers the text-book result again,[16] including the equality $\tilde{\varepsilon} \doteq \varepsilon$ in the geometrical optics approximation. For nonstationary media, the substitution of

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[16] Landau, L. D., and Lifshitz, E. M., Electrodynamics of Continuous Media, (Reading, Massachusetts: Addison-Wesley, 1960, p. 284ff.

(5.12) does not accomplish the same effect owing to the presence of the cross-derivative, $\partial^2/\partial t\partial z$-term in eq. (5.8).* Evidently also, this term reflects an additional coupling effect between medium inhomogeneity and nonstationarity.

Eqs. (5.4) and (5.8) have a somewhat different charactor due to the cross-derivative term in the latter. But the former is closely related to the equation for propagation in a medium with spatially nonuniform gain, viz.

$$\left(\nabla_{\perp}^2 - 2\, ik\, \frac{\partial}{\partial z}\right) e = -ikGe, \qquad (5.15)$$

where $\nabla_{\perp}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $e = e(x,y,z)$ is the envelope emplitude and $G = G(x,y,z)$ is the gain function. Eq. (5.15) has recently been solved analytically, for specified G, by B. N. Perry, et al.,[17] and techniques introduced by these workers should be useful for limiting model forms of eq. (5.4).

Eqs. (5.4) and (5.8) both should be amenable to treatment in a WKB approximation at least, which will shed further light on the physical nature of essential effects associated with medium nonstationary. I plan to do this later.

*The substitution that does it is $B_y = u\tilde{B}_y$, with u being a solution of

$$2\frac{\partial u}{\partial z} + b_1\frac{\partial u}{\partial t} + a_1 u = 0,$$

where a_1 is the coefficient of $\partial B_y/\partial z$ and b_1 the coefficient of $\partial^2 B_y/\partial z\partial t$.

[17]Perry, B. N., Rabinowitz, P., and Newstein, M., Phys. Rev. Lett., Vol. 49, 1982, p. 1921.

CHAPTER 6

SUMMARY AND CONCLUSION

I have derived slow time-scale source-free Maxwell equations for a nonstationary, inhomogeneous medium, assuming a high frequency carrier and slowly varying response functions for the medium. The electric and magnetic complex field amplitude equations are Eq. (4.12) and (4.30).

In nearly all work done on the physics of the coupling of laser energy into fluid, the complex dielectric function $\varepsilon(\omega)$ is assumed spatially uniform and constant. In fact, it is a function of fluid temperature and density, which in many practical cases are functions of position, with steep spatial gradients, and of time. Where the assumptions of homogeneity and stationarity have not been made, as in Ref. 8, a WKB approximation has been used. The equations derived in the present report are designed to permit exploration of effects for energy coupling dynamics of these factors.

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